# Truth Table

A truth table is a way of charting how the Boolean operators (*not*, *and*, *or*) combine.

not = true iff A is false[[1]](#footnote-1)

and = true iff both A and B are true

or = true iff either A is true or B is true

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| possible values | | results of Boolean operators | | | |
| value A | value B | not A | not B | A and B | A or B |
| T | T | F | F | T | T |
| T | F | F | T | F | T |
| F | T | T | F | F | T |
| F | F | T | T | F | F |

To make this more natural, here are some truth tables involving real-word sentences.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| possible values | | results of Boolean operators | | | |
| You have a cat | You have a dog | You do not have a cat | You do not have a dog | You have a cat and a dog | You have a cat or a dog |
| T | T | F | F | T | T |
| T | F | F | T | F | T |
| F | T | T | F | F | T |
| F | F | T | T | F | F |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| possible values | | results of Boolean operators | | | |
| The user wants to save as PDF | The user wants to print | The user does not want to save as PDF | The user does not want to print | The user wants to save as PDF and print | The user wants to save as PDF or print |
| T | T | F | F | T | T |
| T | F | F | T | F | T |
| F | T | T | F | F | T |
| F | F | T | T | F | F |

Notice how in all the truth tables, the pattern of T and F stays exactly the same. This is the power of Boolean operators: they let us generalize the logical value of words regardless of context.

Just for fun, if you turn to the next page, you’ll see what happens when there are more than two sentences…

Let n be the number of distinct sentences. The number of possible T/F assignments is 2n, and the number of and combinations is the nth triangle number.

Here, for example, we have 4 different sentences, A B C D. Thus there are 24 = 16 T/F assignments (rows) and 4 + 3 + 2 + 1 = 10 and combinations (columns). There would also be 10 or combinations, if we had room to show them.[[2]](#footnote-2)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| possible values | | | | results of Boolean operators | | | | | | | | | |
| A | B | C | D | A and B | A and C | A and D | B and C | B and D | C and D | A and B and C | A and B and D | A and C and D | B and C and D |
| T | T | T | T | T | T | T | T | T | T | T | T | T | T |
| T | T | T | F | T | T | F | T | F | F | T | F | F | F |
| T | T | F | T | T | F | T | F | T | F | F | T | F | F |
| T | T | F | F | T | F | F | F | F | F | F | F | F | F |
| T | F | T | T | F | T | T | F | F | T | F | F | T | F |
| T | F | T | F | F | T | F | F | F | F | F | F | F | F |
| T | F | F | T | F | F | T | F | F | F | F | F | F | F |
| T | F | F | F | F | F | F | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | T | T | T | F | F | F | T |
| F | T | T | F | F | F | F | T | F | F | F | F | F | F |
| F | T | F | T | F | F | F | F | T | F | F | F | F | F |
| F | T | F | F | F | F | F | F | F | F | F | F | F | F |
| F | F | T | T | F | F | F | F | F | T | F | F | F | F |
| F | F | T | F | F | F | F | F | F | F | F | F | F | F |
| F | F | F | T | F | F | F | F | F | F | F | F | F | F |
| F | F | F | F | F | F | F | F | F | F | F | F | F | F |

For a final bit of curiosity, if you ever want to construct truth tables of your own, notice how the A B C D columns have a pattern. A alternates T/F every 2n-1 = 8 rows; B alternates T/F every 2n-2 = 4 rows; C alternates T/F every 2n-3 = 2 rows; and D alternates T/F every 2n-4 = 1 row. This is an easy way to write your initial T/F values while being sure you’ve hit every possible combination.

1. ‘iff’ is a common term in math & computer science that is shorthand for ‘if and only if’, which means we’re saying we have stated every possible cause, excluding all other causes. For example, ‘You may skip the line iff you are the president’ means that the president can skip, *and* no one elsebesides the president can skip. [↑](#footnote-ref-1)
2. I really should have made this in Excel. It took way too long to type and shade these. Formulas and conditional formatting would have done it instantly. In fact, [here’s a spreadsheet where I did that](https://docs.google.com/spreadsheets/d/1OVcBl_ZGzvLbkDzBbVC-R_-XfBPDduzsioOd7MzGViI/edit?usp=sharing)… [↑](#footnote-ref-2)